ISSN: 2581-3064
sjrmcseditor@scischolars.com Online Publication Date: July 15, 2017 Volume 2, No. 1

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Scholars Journal of Research in Mathematics and Computer Science

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A Study of Magnetoelastic Rayleigh Wave Propagation in a Three Layered Medium

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Abstract

The propagation of Rayleigh-type waves in a three-layered medium is studied analytically in the context of linear magnetoelasticity. The intermediate layer is one of finite thickness, bounded on both sides by layers of very large depths,all three layers assumed to be perfect conductors of electricity. Dispersive Rayleigh-waves are found to exist under an initially uniform magnetic field transverse to the displacement components. Numerical studies show the effect of the field intensity, and the depth of the middle-layer on the velocity of the propagating waves.

Keywords: Magnetoelasticity; Perfect conductivity; Phase Velocity; Wave Propagation.

1. Introduction

The theory of surface wave propagation in conducting solids under the influence of external magnetic field is of immense interest in view of possible applications in both geophysics and seismology as well as electronics and telecommunications. Such seismic waves propagating under the influence of earth magnetic field have been studied by Knopoff [1], Kalishky and Rogula [2], Tomita and Shindo [3] and others. Such waves can be applied for nondestructive evaluation of physical parameter and also used for electromagnetic sensors and other devices. Some recent studied on surface wave propagation in conducting materials include Viktorov[4] ,Lee and Its[5], Chakraborty and Chattopadhyay [6] , Barik and Chakraborty [7], Chakraborty and Barik [8].

In many situations it is necessary to consider more than one layer for example while considering propagation of seismic surface waves.Different velocity gradient in the crust or mantle can be approximated by several homogeneous layers(Ewing and Jardetzky [9]. Rayleigh waves are this type expected to be dispersive. Love wave in a three layer mediums have been discussed by Stoneley[10].

Recently this problem has to be extended by Chakraborty and Barik [11] to three layer structure consisting of different perfectly conducting medium where waves were seen to exist even in those cases where ordinary elastic waves are absent due to the presence of a magnetic field. Rayleigh waves in a three layer medium have been studied by Sinha [12] and Dutta and Roy [13].

This paper is concerned with the analytical study of magnetoelastic Rayleigh-type wave propagation in a structure comprising of two semi-spaces and one layer of finite depth in between. The initial magnetic field is taken transverse to the displacement components. It is seen that such waves exist and propagate with a velocity which is less than the S-wave velocities in all the three materials comprising the medium. However, classical waves of this type does not exist if the depth of the middle-layer is small, but the presence of the magnetic field will allow such waves to propagate even for thin layers. The velocity profile has been shown for different cases. Numerical investigation show the effect of the magnetic field on the wave velocity for first mode of the dispersive waves. As the field intensity is increase waves with larger wave lengths come into existence.

2. Formulation of the Problem

We consider a perfectly conducting isotropic layer M_2 of thickness d sandwiched between two perfectly conducting isotropic half spaces M_1 and M_3. The origin has been taken at the upper interface, Rayleigh wave propagates toward x-axis, while the positive z-axis toward the interior of the lower half space. The semi-spaces M_1 and M_3 are represented by $z \leq 0$ and $z \geq d$ respectively and the layer M_2 occupies the region $0 \leq z \leq d$. The Lame's constant, density and magnetic permeability are given by (λ_α, G_α), ρ_α and μ_α respectively with α = 1 for M_1, α = 2 for M_2 and α =3 for M_3. σ is electrical conductivity for M_2.

The motion is governed by the following equations for magnetoelastic disturbances :

(I). Maxwell's equations of the electromagnetic field:

$$\nabla . \mathbf{D} = 0; \ \nabla . \mathbf{B} = 0;$$

$$\nabla \times \mathbf{H} = \mathbf{J}; \ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}}; \tag{1}$$

(displacement current is neglected)
where **D** is the electric displacement, **B** is the magnetic induction, **H** is the magnetic field, **E** is electric field and **J** is current density.

(II). The equations of motion:

$$\tau_{ij,j} + [\mathbf{J} \times \mathbf{B}]_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \ i, j = 1, 2, 3, \tag{2}$$

Where $\mathbf{J} \times \mathrm{B}$ is the Lorenz force due to the electromagnetic field and τ_{ij} being the usual elastic stress tensor in the medium.

(III). The constitutive equations:

$$\mathbf{B}=\mu\mathbf{H}; \ \mathbf{D}=\varepsilon\mathbf{E}; \tag{3}$$

with Ohm's law as

$$\mathbf{J} = \sigma(\mathbf{E} + \dot{\mathrm{u}} \times \mathbf{B}), \tag{4}$$

where σ is the electric conductivity, μ is the magnetic permeability, $\dot{\mathbf{U}}$ the particle velocity and ϵ the permittivity.

(IV). Elastic Stress-Strain relations:

$$\tau_{ij} = \lambda u_{k,k} \delta_{i,j} + 2Ge_{ij}, \tag{5}$$

where e_{ij} are the strain components and

$$2e_{ij} = u_{i,j} + u_{j,i} \ i, j= 1,2,3, \tag{6}$$

u_i being particle displacement and λ, G are Lame constants.

(V). The electromagnetic boundary conditions:

$$\mathbf{n}.||\mathbf{B}||=0, \quad \mathbf{n}\times||\mathbf{H}||=0, \quad \mathbf{n}\times||\mathbf{E}+\dot{\mathbf{u}}\times\mathbf{B}||=0.. \tag{7}$$

where n is normal to the interfaces or surface z = constant and ||.|| discontinuity jump across the boundary of the vector within.

(VI). Stress continuity conditions across the boundaries z = constant:

$$\left(\tau_{3j} + \tau^{E}{}_{3j}\right)_+ - \left(\tau_{3j} + \tau^{E}{}_{3j}\right)_- = 0, \ j = 1,2,3. \tag{8}$$

where $\tau^E{}_{ij}$ are Maxwell's stress tensor components due to a magnetic field given by $\tau^E{}_{ij} = \mu(H_i h_j + H_j h_i - H_k h_k \delta_{i,j})$.

There is no initial electric field and the current density **J** is initially zero. The displacement current **D**, being small, is neglected throughout this problem.

We consider the wave propagation in x-direction and displacement field

$$\mathbf{u}^{(\alpha)} = \left(u_\alpha{}^{(\alpha)}(x,z,t), 0, w_\alpha{}^{(\alpha)}(x,z,t)\right), \tag{9}$$

with α = 1 for M_1, α = 2 for M_2 and α =3 for M_3.

The bias magnetic field $(0, H_2{}^{(\alpha)}, 0)$ with α = 1 for M_1, α = 2 for M_2 and α =3 for M_3.

The perturbed magnetic field are

$$\left(h_1{}^{(\alpha)}, H_2{}^{(\alpha)} + h_2{}^{(\alpha)}, h_3{}^{(\alpha)}\right), \tag{10}$$

with α = 1 for M_1 , α = 2 for M_2 respectively.

Use electro magnetic boundary conditions (7), we have

$$H_2{}^{(1)} = H_2{}^{(2)} = H_2{}^{(3)} = H_2 \text{ (say)} \tag{11}$$

$$\text{Hence, } \mathbf{H}^{(1)} = \mathbf{H}^{(2)} = \mathbf{H}^{(3)} = (0, H_2, 0). \tag{12}$$

For a perfectly conducting medium $\sigma \to \infty$ and (4) gives the perturbed electric field, which substituting in (1) gives

$$\frac{\partial \mathbf{h}^{(\alpha)}}{\partial t} = \left(0, -H_2{}^{(\alpha)}\frac{\partial^2 w^{(\alpha)}}{\partial z\,\partial t} - H_2{}^{(\alpha)}\frac{\partial^2 u^{(\alpha)}}{\partial x\,\partial t}, 0\right). \tag{13}$$

(neglecting second order terms)

Integrating (13) with respect to time t

$$\mathbf{h}^{(\alpha)} = \left(0, -H_2{}^{(\alpha)}\frac{\partial w^{(\alpha)}}{\partial z} - H_2{}^{(\alpha)}\frac{\partial u^{(\alpha)}}{\partial x}, 0\right) \tag{14}$$

The Lorenz force is

$$\mathbf{J}^{(\alpha)} \times \mathbf{B}^{(\alpha)} = \left(-\mu_\alpha H_2{}^{(\alpha)} h_{2_x}{}^{(\alpha)}, 0, -\mu_\alpha H_2{}^{(\alpha)} h_{2_z}{}^{(\alpha)}\right). \tag{15}$$

(neglecting second order terms)

The equations of motion are

$$(\alpha_1{}^2 + a_1{}^2)u_{xx}{}^{(1)} + b_1{}^2 u_{zz}{}^{(1)} + (\alpha_1{}^2 - b_1{}^2 + a_1{}^2)w_{xz}{}^{(1)} = u_{tt}{}^{(1)}, \tag{16}$$

$$(\alpha_1{}^2 - b_1{}^2 + a_1{}^2)u_{xz}{}^{(1)} + b_1{}^2 w_{xx}{}^{(1)} + (\alpha_1{}^2 + a_1{}^2)w_{zz}{}^{(1)} = w_{tt}{}^{(1)} \text{ in } M_1 \tag{17}$$

$$(\alpha_2{}^2 + a_2{}^2)u_{xx}{}^{(2)} + b_2{}^2 u_{zz}{}^{(2)} + (\alpha_2{}^2 - b_2{}^2 + a_2{}^2)w_{xz}{}^{(2)} = u_{tt}{}^{(2)}, \tag{18}$$

$$(\alpha_2{}^2 - b_2{}^2 + a_2{}^2)u_{xz}{}^{(2)} + b_2{}^2 w_{xx}{}^{(2)} + (\alpha_2{}^2 + a_2{}^2)w_{zz}{}^{(2)} = w_{tt}{}^{(2)} \text{ in } M_2 \tag{19}$$

$$(\alpha_3{}^2 + a_3{}^2)u_{xx}{}^{(3)} + b_3{}^2 u_{zz}{}^{(3)} + (\alpha_3{}^2 - b_3{}^2 + a_3{}^2)w_{xz}{}^{(3)} = u_{tt}{}^{(3)}, \tag{20}$$

$$(\alpha_3{}^2 - b_3{}^2 + a_3{}^2)u_{xz}{}^{(3)} + b_3{}^2 w_{xx}{}^{(3)} + (\alpha_3{}^2 + a_3{}^2)w_{zz}{}^{(3)} = w_{tt}{}^{(3)} \text{ in } M_3 \tag{21}$$

$$\text{Where } a_i{}^2 = \frac{\mu_i H_2{}^2}{\rho_i};\ b_i{}^2 = \frac{G_i}{\rho_i};\ \alpha_i{}^2 = \frac{\lambda_i + 2G_i}{\rho_i} \tag{22}$$

a_i is Alfven wave velocity, α_i is P-wave velocity and b_i is S-wave velocity in M_i, for i = 1,2,3.

Introducing

$$x' = kx;\ z' = kz;\ t' = kb_2 t;\ u'^{(i)} = ku^{(i)};\ w'^{(i)} = kw^{(i)};\ c' = \frac{c}{b_2};\ \text{for I} = 1,2,3 \quad . \tag{23}$$

(where k is the wave number)

Using (23), the equations (16) - (21) reduces to

$$(\alpha_1{}^2 + a_1{}^2)u'^{(1)}{}_{x'x'} + b_1{}^2 u'^{(1)}{}_{z'z'} + (\alpha_1{}^2 - b_1{}^2 + a_1{}^2)w'^{(1)}{}_{x'z'} = b_2{}^2 u'^{(1)}{}_{t't'}, \tag{24}$$

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$$({\alpha_1}^2 - {b_1}^2 + {a_1}^2)u'^{(1)}_{x'z'} + {b_1}^2 w'^{(1)}_{x'x'} + ({\alpha_1}^2 + {a_1}^2)w'^{(1)}_{z'z'} = {b_2}^2 w'^{(1)}_{t't'}, \tag{25}$$

$$({\alpha_2}^2 + {a_2}^2)u'^{(2)}_{x'x'} + {b_2}^2 u'^{(2)}_{z'z'} + ({\alpha_2}^2 - {b_2}^2 + {a_2}^2)w'^{(2)}_{x'z'} = {b_2}^2 u'^{(2)}_{t't'}, \tag{26}$$

$$({\alpha_2}^2 - {b_2}^2 + {a_2}^2)u'^{(2)}_{x'z'} + {b_2}^2 w'^{(2)}_{x'x'} + ({\alpha_2}^2 + {a_2}^2)w'^{(2)}_{z'z'} = {b_2}^2 w'^{(2)}_{t't'}, \tag{27}$$

$$({\alpha_3}^2 + {a_3}^2)u'^{(3)}_{x'x'} + {b_3}^2 u'^{(3)}_{z'z'} + ({\alpha_3}^2 - {b_3}^2 + {a_3}^2)w'^{(3)}_{x'z'} = {b_2}^2 u'^{(3)}_{t't'}, \tag{28}$$

$$({\alpha_3}^2 - {b_3}^2 + {a_3}^2)u'^{(3)}_{x'z'} + {b_3}^2 w'^{(3)}_{x'x'} + ({\alpha_3}^2 + {a_3}^2)w'^{(3)}_{z'z'} = {b_2}^2 w'^{(3)}_{t't'}, \tag{29}$$

2.1 Boundary Condition

At interface stress are continuous at $z' = 0$. Then from (8) we have

$$[\,G_1(u'^{(1)}_{z'} + w'^{(1)}_{x'})]_{z' \to 0^-} = [\,G_2(u'^{(2)}_{z'} + w'^{(2)}_{x'})]_{z' \to 0^+}, \tag{30a}$$

$$[(\lambda_1 + 2G_1)\,w'^{(1)}_{z'} + \lambda_1 u'^{(1)}_{x'}]_{z' \to 0^-} = [(\lambda_2 + 2G_2)w'^{(2)}_{z'} + \lambda_2 u'^{(2)}_{x'}]_{z' \to 0^+}. \tag{30b}$$

At interface displacement being continuous at $z' = 0$. Then we have

$$[(u'^{(1)},\ 0,\ w'^{(1)})]_{z' \to 0^-} = [(u'^{(2)},\ 0,\ w'^{(2)})]_{z' \to 0^+}. \tag{30c}$$

At interface stress are continuous at $z' = kd$. Then from (8) we have

$$[\,G_2(u'^{(2)}_{z'} + w'^{(2)}_{x'})]_{z' \to kd^-} = [\,G_3(u'^{(3)}_{z'} + w'^{(3)}_{x'})]_{z' \to kd^+}, \tag{30d}$$

$$[(\lambda_2 + 2G_2)\,w'^{(2)}_{z'} + \lambda_2 u'^{(2)}_{x'}]_{z' \to kd^-} = [(\lambda_3 + 2G_3)w'^{(3)}_{z'} + \lambda_3 u'^{(3)}_{x'}]_{z' \to kd^+}. \tag{30e}$$

At interface displacement being continuous at $z' = kd$. Then we have

$$[(u'^{(2)},\ 0,\ w'^{(2)})]_{z' \to 0^-} = [(u'^{(3)},\ 0,\ w'^{(3)})]_{z' \to 0^+}. \tag{30f}$$

3. Wave Solution

Solution of equation (24) - (29) are assumed in the form

$$(u'^{(i)}, 0, w'^{(i)}) = (\hat{u}'^{(i)}(z'), 0, \hat{w}'^{(i)}(z'))e^{i(x' - c' t')} \tag{31}$$

where $\hat{u}'^{(i)}(z'), \hat{w}'^{(i)}(z')$) are functions of z' only and i = 1 for M_1, i = 2 for M_2 and i =3 for M_3.

Substituting (31) into the equations (24) - (25), we take exponential solutions of the form

$$u'^{(1)}(x', z', t') = (A_{11}e^{p_1 z'} + A_{12}e^{q_1 z'})\, e^{i(x' - c' t')}, \tag{32}$$

$$w'^{(1)}(x', z', t') = (A'_{11}e^{p_1 z'} + A'_{12}e^{q_1 z'})\, e^{i(x' - c' t')}, \tag{33}$$

where $A_{11}, A_{12}, A'_{11}, A'_{12}$ are constants and

$$p_1 = \sqrt{1 - \frac{{s_1}^2 {c'}^2}{\frac{{\alpha_1}^2}{{b_1}^2} + \frac{{a_1}^2}{{b_1}^2}}},\ q_1 = \sqrt{1 - {s_1}^2 {c'}^2},\quad {s_1}^2 = \frac{{b_2}^2}{{b_1}^2}. \tag{34}$$

Using (32) and (33) in (24) and (25), we obtain to zero, we obtain

$$A'_{11} = k_{11}A_{11},\ A'_{12} = k_{12}A_{12}, \tag{35}$$

$$\text{Where } k_{11} = -ip_1,\ k_{12} = -\frac{i}{q_1}. \tag{36}$$

Finally the displacement components for the medium M_1 may be expressed as

$$u'^{(1)}(x', z', t') = (A_{11}e^{p_1 z'} + A_{12}e^{q_1 z'})\, e^{i(x' - c' t')}, \tag{37}$$

$$w'^{(1)}(x', z', t') = (k_{11}A_{11}e^{p_1 z'} + k_{12}A_{12}e^{q_1 z'})\, e^{i(x' - c' t')}. \tag{38}$$

Similarly, for the medium M_2, we derive

ISSN: 2581-3064
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$$u'^{(2)}(x',z',t') = \left(A_{21}e^{p_2 z'} + A_{22}e^{-p_2 z'} + A_{23}e^{q_2 z'} + A_{24}e^{-q_2 z'}\right) e^{i(x' - c' t')}, \tag{39}$$

$$w'^{(2)}(x',z',t') = \left(k_{21}A_{21}e^{p_2 z'} + k_{22}A_{22}e^{-p_2 z'} + k_{23}A_{23}e^{q_2 z'} + k_{24}A_{24}e^{-q_2 z'}\right) e^{i(x' - c' t')}, \tag{40}$$

where $A_{21}, A_{22}, A_{24}, A_{24}$ are constants and

$$p_2 = \sqrt{1 - \frac{c'^2}{\frac{\alpha_2^2}{b_2^2} + \frac{a_2^2}{b_2^2}}}, \quad q_2 = \sqrt{1 - c'^2}, \quad k_{21} = -ip_2, k_{22} = ip_2, \quad k_{23} = -\frac{i}{q_2}, \quad k_{24} = \frac{i}{q_2}. \tag{41}$$

Similarly, for the medium M_2, we derive

$$u'^{(3)}(x',z',t') = \left(A_{31}e^{-p_3 z'} + A_{32}e^{-q_3 z'}\right) e^{i(x' - c' t')}, \tag{42}$$

$$w'^{(3)}(x',z',t') = \left(k_{31}A_{31}e^{-p_3 z'} + k_{32}A_{32}e^{-q_3 z'}\right) e^{i(x' - c' t')}, \tag{43}$$

where A_{31}, A_{32} are constants and

$$p_3 = \sqrt{1 - \frac{s_3^2 c'^2}{\frac{\alpha_3^2}{b_3^2} + \frac{a_3^2}{b_3^2}}}, \quad q_3 = \sqrt{1 - s_3^2 c'^2}, \quad s_3^2 = \frac{b_2^2}{b_3^2}, \quad k_{31} = ip_3, \quad k_{32} = \frac{i}{q_2}. \tag{44}$$

The conditions (30a - 30f) are used to eliminate the constants $A_{11}, A_{12}, A_{21}, A_{22}, A_{24}, A_{24}, A_{31}, A_{32}$ resulting in the frequency equation

$$\begin{vmatrix} 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 \\ p_1 & 1/q_1 & -p_2 & p_2 & -1/q_2 & 1/q_2 & 0 & 0 \\ 2p_1 & q_1 + 1/q_1 & -2s_1^2\rho'_1 p_2 & 2s_1^2\rho'_1 p_2 & -s_1^2\rho'_1(q_2 + 1/q_2) & s_1^2\rho'_1(q_2 + 1/q_2) & 0 & 0 \\ m_{11} & -2 & -s_1^2\rho'_1 m_{22} & -s_1^2\rho'_1 m_{22} & 2s_1^2\rho'_1 & 2s_1^2\rho'_1 & 0 & 0 \\ 0 & 0 & e^{kp_2d} & e^{-kp_2d} & e^{kq_2d} & e^{-kq_2d} & 1 & 1 \\ 0 & 0 & -p_2e^{kp_2d} & p_2e^{-kp_2d} & -e^{kq_2d}/q_2 & e^{-kq_2d}/q_2 & p_3 & 1/q_3 \\ 0 & 0 & 2s_3^2\rho'_3 p_2 e^{kp_2d} & -2s_3^2\rho'_3 p_2 e^{-kp_2d} & s_3^2\rho'_3(q_2 + 1/q_2)e^{kq_2d} & -s_3^2\rho'_3(q_2 + 1/q_2)e^{-kq_2d} & 2p_3 & q_3 + \frac{1}{q_3} \\ 0 & 0 & s_3^2\rho'_3 m_{22} e^{kp_2d} & s_3^2\rho'_3 m_{22} e^{-kp_2d} & -2s_3^2\rho'_3 e^{kq_2d} & -2s_3^2\rho'_3 e^{-kq_2d} & m_{33} & -2 \end{vmatrix} = 0, \tag{45}$$

where $m_{11} = \frac{\alpha_1^2}{b_1^2}(1 - p_1^2) - 2; \;\; m_{22} = \frac{\alpha_2^2}{b_2^2}(1 - p_2^2) - 2; \;\; m_{33} = \frac{\alpha_3^2}{b_3^2}(1 - p_3^2) - 2.$ (46)

In absence of any magnetic field, this frequency equation agrees with that obtained by Dutta and Roy [13].

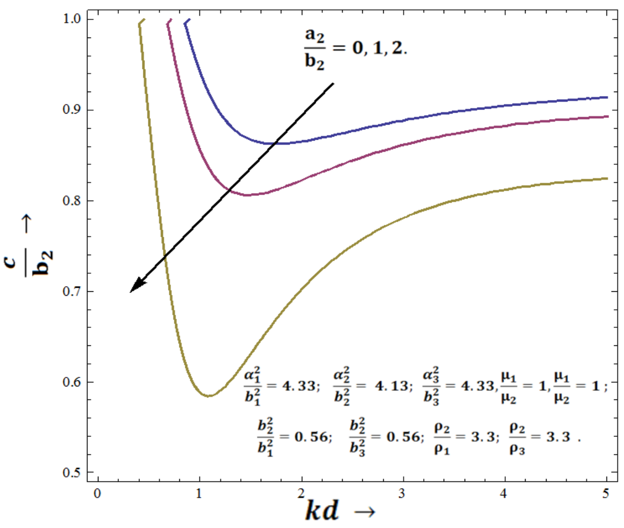

Fig 1:

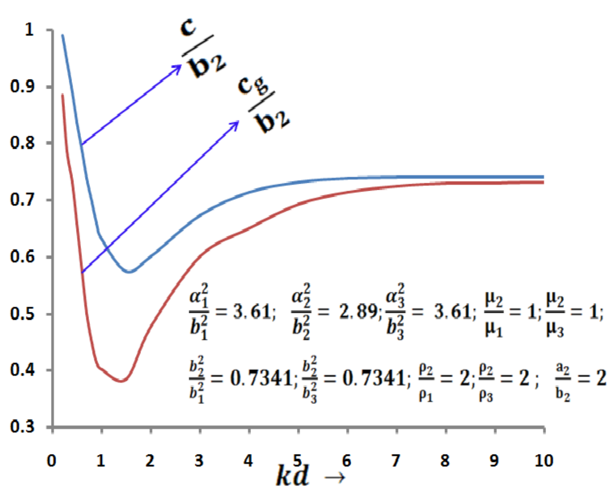

Fig 2:

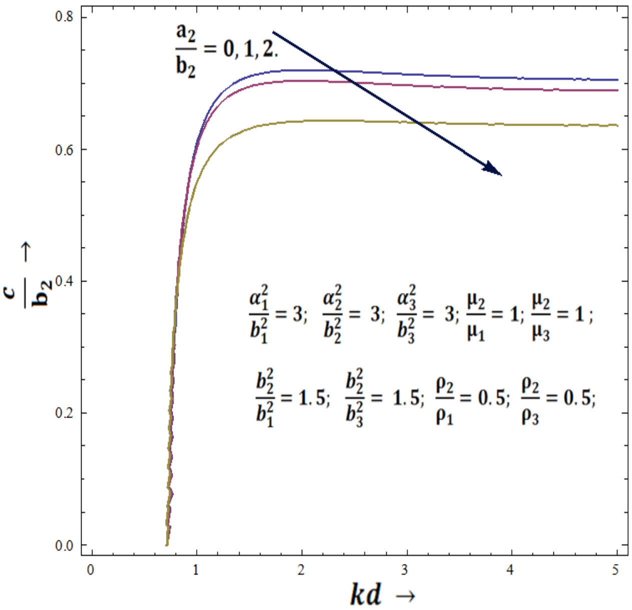

Fig 3:

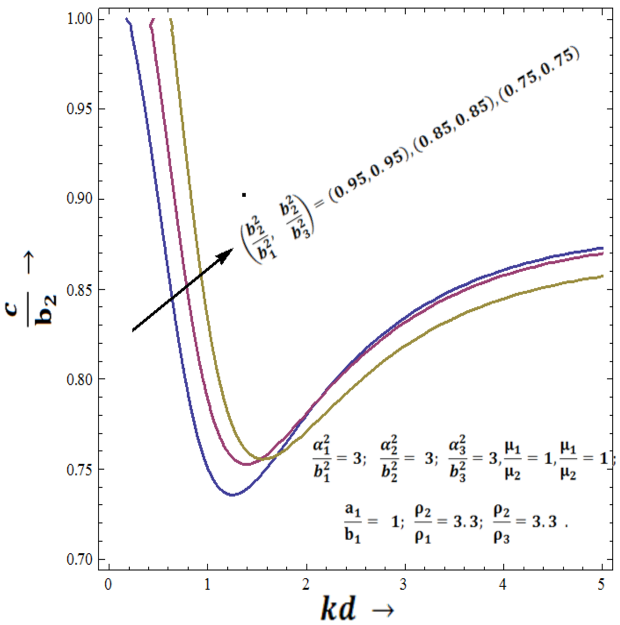

Fig 4:

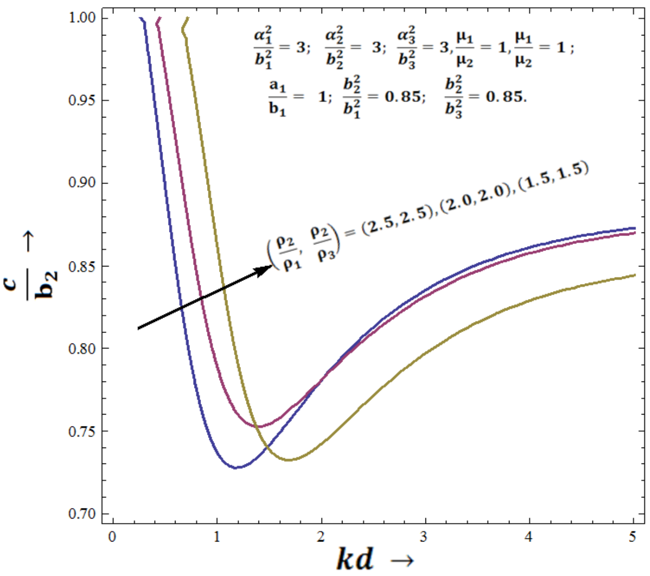

Fig 5:

4. Numerical Results

The frequency equation (45) has been solved numerically for the case $b_2 < b_1, b_3$ and also for $b_2 > b_1, b_3$ .These are dispersive waves, the non-dimensional wave velocity have been plotted against non-dimensional depth for first modes. Figures(1) and (2) show the variation in wave velocity and dispersion effect for a Copper-Silver-Copper medium $b_2 < b_1, b_3$). Waves are seen to exist even for lower values of kd (non-dimensional depth) when the magnetic field is high (by increasing a_2). This implies for a fixed depth waves with lower wave numbers i.e. long waves exist only when the field is suitable high. In figures(4) and(5)the wave velocity is plotted against kd for a large number of materials by varying the ratio of physical parameters but always keeping the slowest medium in the middle. Figure (3)shows the velocity when the faster medium in the middle. Here the only effect of the magnetic field is to increase the wave velocity.

5. Conclusion

The Rayleigh-wave propagation in a three layer medium is significant principally because it can be used to understand the propagation of seismic waves in the mantle and crust influenced by the earth's magnetic field. The results described above can be applied to testing of materials parameters of advance solid structures. It has been established that such waves may propagate when the slower medium is bounded by two faster media. Waves loose their existence for a very thin layer although imposition of a magnetic field might induce longer waves to exist. Waves of shorter waves length travel with a velocity which decreases with increasing magnetic field.

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